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Equidistribution of Hecke eigenforms on the Hilbert modular varieties

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Abstract

Let F be a totally real number field with ring of integers $\mathcal{O}$, and let $\Gamma = SL(2, \mathcal{O})$ be the Hilbert modular group. Given the orthonormal basis of Hecke eigenforms in $S_{2k}(\Gamma)$, one can associate a probability measure $d\mu_k$ on the Hilbert modular variety $\Gamma\backslash\mathbb{H}^n$. We prove that $d\mu_k$ tends to the invariant measure on $\Gamma\backslash\mathbb{H}^n$ weakly as $k \to \infty$. This generalizes Luo's result [W. Luo, Equidistribution of Hecke eigenforms on the modular surface, Proc. Amer. Math. Soc. 131 (2003) 21–27] for the case $F = Q$.

Keywords: Hilbert modular form; Hecke eigenform; Bergman kernel

1. Introduction

Let F be a totally real number field of degree n over $\mathbb{Q}$ with ring of integers $\mathcal{O}$ and $\sigma_1, \sigma_2, \ldots, \sigma_n$ be all the real embeddings of F. Let $\Gamma = SL(2, \mathcal{O})$ be the Hilbert modular group which acts discontinuously on the product of n upper half planes $\mathbb{H}^n$ in the following way: For $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \Gamma$, and $z = (z_1, \ldots, z_n) \in \mathbb{H}^n$, we define $\gamma z = (\gamma_1 z_1, \ldots, \gamma_n z_n)$ where

$$\gamma_i = \begin{pmatrix} \sigma_i(a) & \sigma_i(b) \\ \sigma_i(c) & \sigma_i(d) \end{pmatrix}, \qquad \gamma_i z_i = \frac{\sigma_i(a) z_i + \sigma_i(b)}{\sigma_i(c) z_i + \sigma_i(d)} \quad (1 \leqslant i \leqslant n).$$

Remark. We may also identify Γ with its image in $SL(2, \mathbb{R})^n$ via $\gamma \in \Gamma$, $\gamma = (\gamma_1, \ldots, \gamma_n) \in SL(2, \mathbb{R})^n$.

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It is well known that Γ has finite co-volume (see [Fr]), i.e.

$$\mathrm{vol}\big(\Gamma\backslash\mathbb{H}^n\big)=\int\limits_{\Gamma\backslash\mathbb{H}^n}\frac{dx\,dy}{(Ny)^2}<\infty,$$

where $z=(x_1+iy_1,\dots,x_n+iy_n)\in\mathbb{H}^n$, $dx=dx_1\cdots dx_n$, $dy=dy_1\cdots dy_n$, and $Ny=y_1\cdots y_n$.

Denote by $S_{2k}(\Gamma)$ ($k\in\mathbb{N}$, $k\geqslant 2$) the space of Hilbert modular cusp forms of weight $(2k,\dots,2k)$, i.e. the space of holomorphic functions $f(z)$ on $\mathbb{H}^n$ such that

(1) for $\gamma=\left(\begin{smallmatrix}a&b\\c&d\end{smallmatrix}\right)\in\Gamma$, $f(\gamma z)=N(cz+d)^{2k}f(z)$, where for $z=(z_1,\dots,z_n)\in\mathbb{H}^n$,

$$N(cz+d)=\prod_{i=1}^{n}\big(\sigma_i(c)z_i+\sigma_i(d)\big),$$

(2) $f(z)$ vanishes at all the cusps of Γ (see [Ga] or [Fr]).

Let

$$d\mu=\frac{1}{\mathrm{vol}(\Gamma\backslash\mathbb{H}^n)}\frac{dx\,dy}{(Ny)^2}.$$

For f and g in $S_{2k}(\Gamma)$, we define the (normalized) Petersson inner product by

$$\langle f,g\rangle=\int\limits_{\Gamma\backslash\mathbb{H}^n}f(z)\overline{g(z)}(Ny)^{2k}\,d\mu.$$

It is well known that $S_{2k}(\Gamma)$ is a finite dimensional Hilbert space. Furthermore, if we let $J_k=\dim_{\mathbb{C}}S_{2k}(\Gamma)$, then it was shown by Shimizu [Sh] (using the Selberg trace formula) that

$$J_k=\frac{\mathrm{vol}(\Gamma\backslash\mathbb{H}^n)}{(4\pi)^n}(2k-1)^n+O(1) \tag{1.1}$$

as $k\to\infty$.

One expects the following mass equidistribution conjecture on the Hilbert modular variety $\Gamma\backslash\mathbb{H}^n$ should be true:

$$\lim_{k\to\infty}\max_{1\leqslant i\leqslant J_k}\left|\int\limits_A(Ny)^{2k}\big|f_{i,k}(z)\big|^2\,d\mu-\int\limits_A d\mu\right|=0 \tag{1.2}$$

where $A\subset\Gamma\backslash\mathbb{H}^n$ is compact and $\{f_{i,k}\}_{i=1}^{J_k}$ is the orthonormal Hecke basis of $S_{2k}(\Gamma)$. For $n=1$ (i.e. $\Gamma=\Gamma(1)$), this is an analogue of quantum unique ergodicity conjecture, formulated by Rudnick and Sarnak [RS].

This conjecture is still out of reach at the present. However, Luo [Lu] established this conjecture on the average and Lau [La] generalized Luo's result to the arithmetic surface $\Gamma_0(N)\backslash\mathbb{H}$. The purpose of this paper is to generalize Luo's and Lau's results to the Hilbert modular varieties (Theorem 1 and Corollary 2).

Let $\{f_{i,k}\}_{i=1}^{J_k}$ be an orthonormal basis of $S_{2k}(\Gamma)$. Set

$$d\mu_k = \frac{1}{J_k}\left(\sum_{i=1}^{J_k}\left|f_{i,k}(z)\right|^2\right)(Ny)^{2k}d\mu.$$

Theorem 1. *For any compact subset* $A \subset \Gamma\backslash\mathbb{H}^n$ *and any* $0 < \epsilon < 1$, *we have*

$$\int_A d\mu_k = \int_A d\mu + O_{\epsilon,A}\big(\big(k^{-1+\epsilon}\big)^n\big)$$

as $k \to \infty$.

Remark 1. The key ingredients in [Lu] and [La] are the Bergman kernel for the Hecke operator and the Petersson trace formula, respectively. Our approach is using the Bergman kernel on $\Gamma\backslash\mathbb{H}^n$.

Remark 2. Luo [Lu] proved a uniform result for all measurable subsets A. In our Theorem 1, the result depends on the compact subset A. But our decay rate is sharper than in [Lu].

Some properties of Γ. We say that an element γ ($\neq$ *identity*) of Γ is *elliptic* (respectively *parabolic* and *hyperbolic*) if all the γ_i are elliptic (respectively parabolic and hyperbolic) in the usual sense (see [Iw]). If γ ($\neq$ *identity*) is not of above types, we say that γ is *mixed*. A point z in $\mathbb{H}^n$ is called an *elliptic point* if it is fixed by an elliptic element in Γ. A point κ in $\overline{\mathbb{R}}^n$ (where $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$) is called a *cusp* if it is fixed by a parabolic element in Γ.

Proposition 1. *(See [Sh, Theorem 6].) The number of the* Γ*-inequivalent elliptic points of* Γ *is finite.*

Proposition 2. *(See [Sh, Lemma 15].) Let* $e_1, \ldots, e_s \in \mathbb{H}^n$ *be a complete representatives of* Γ*-inequivalent elliptic points of* Γ*. Then the union of* $\Gamma_{e_i} \setminus \{1\}$ $(1 \leqslant i \leqslant s)$ *forms a complete representatives of non-conjugate elliptic elements in* Γ*, where* $\Gamma_{e_i} = \{\gamma \in \Gamma\colon \gamma e_i = e_i\}$ $(1 \leqslant i \leqslant s)$.

Since Γ_{e_i} is a discrete subgroup of a compact subgroup, Γ_{e_i} is a finite subgroup. Hence we have

Lemma 1. *There are only finitely many elliptic conjugacy classes in* Γ.

2. Bergman kernel

For $k \in \mathbb{N}$, $k \geqslant 2$ and $z = (z_1, \ldots, z_n)$, $w = (w_1, \ldots, w_n) \in \mathbb{H}^n$, we define the Bergman kernel by

$$B_k(z, w) = \sum_{\gamma \in \Gamma} N(\gamma z - \overline{w})^{-2k} j(\gamma, z)^{-2k}$$

where $N(\gamma z - \overline{w}) = \prod_{i=1}^n (\sigma_i(\gamma) z_i - \overline{w_i})$ and $j(\gamma, z) = N(cz + d)$, $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$.

Proposition 3.

(1) *$B_k(z,w)$ converges absolutely and uniformly for (z,w) in compact subsets of $\mathbb{H}^n \times \mathbb{H}^n$.*
(2) *For each fixed $w \in \mathbb{H}^n$, $B_k(z,w) \in S_{2k}(\Gamma)$ (as a function of z).*

Proof. The proof can be found in [Ga, 1.14] or [Fr, Chapter II]. □

Proposition 4. *If $f \in S_{2k}(\Gamma)$, then*

$$f(w) = \left(\frac{2k-1}{4\pi}\right)^n \frac{(2i)^{2kn}}{2} \int\limits_{\Gamma\backslash\mathbb{H}^n} f(z)\overline{B_k(z,w)}(Ny)^{2k}\frac{dx\,dy}{(Ny)^2}$$
$$= \left(\frac{2k-1}{4\pi}\right)^n \frac{(2i)^{2kn}}{2} \operatorname{vol}\bigl(\Gamma\backslash\mathbb{H}^n\bigr)\bigl\langle f, B_k(\cdot,w)\bigr\rangle$$

where $z = (x_1+iy_1,\dots,x_n+iy_n) \in \mathbb{H}^n$, $w \in \mathbb{H}^n$.

Proof. See [Ga, 1.14] or [Fr, Chapter II]. □

For convenience, denote by

$$C_k^{-1} = \left(\frac{2k-1}{4\pi}\right)^n \frac{(2i)^{2kn}}{2} \operatorname{vol}\bigl(\Gamma\backslash\mathbb{H}^n\bigr) \tag{2.1}$$

and note that $C_k = \overline{C_k}$ when $k \geqslant 2$.

For $k \in \mathbb{N}$, $\gamma \in \Gamma$ and $z = (z_1,\dots,z_n) \in \mathbb{H}^n$, let

$$h(\gamma,z) = N(z-\bar z)^2 N(\gamma z - \bar z)^{-2} j(\gamma,z)^{-2}$$

and

$$h_k(\gamma,z) = \bigl(h(\gamma,z)\bigr)^k = N(z-\bar z)^{2k} N(\gamma z-\bar z)^{-2k} j(\gamma,z)^{-2k}.$$

Lemma 2. *$|h_k(\gamma,z)| \leqslant 1$ for all $z \in \mathbb{H}^n$ and $\gamma \in \Gamma$. Moreover, $|h_k(\gamma,z)| = 1$ if and only if $\gamma = \pm 1$ or γ is elliptic and z is its fixed point.*

Proof. It suffices to prove when $n = 1$. By definition,

$$\bigl|h_k(\gamma,z)\bigr| = \left|\frac{z-\bar z}{\gamma z - \bar z}\cdot\frac{1}{cz+d}\right|^{2k}$$

where $\gamma = \left(\begin{smallmatrix} a & b \\ c & d\end{smallmatrix}\right) \in \Gamma$. Let $\gamma z = z' = x' + iy'$ and $z = x+iy$. Then

$$\left|\frac{z-\bar z}{\gamma z-\bar z}\cdot\frac{1}{cz+d}\right| = \frac{y^{1/2}}{\left|\frac{(x'-x)+i(y+y')}{2i}\right|}\left(\frac{y}{|cz+d|^2}\right)^{1/2}$$

$$= \frac{y^{1/2}(y')^{1/2}}{\left|\frac{y+y'}{2} + i\frac{x-x'}{2}\right|} \leqslant \frac{y^{1/2}(y')^{1/2}}{\frac{y+y'}{2}} \leqslant 1.$$

The equality holds if and only if $x = x'$ and $y = y'$, i.e. $\gamma z = z$. Hence the equality holds if and only if $\gamma = \pm 1$ or γ is elliptic and z is its fixed point. □

Lemma 3. *For each fixed $k \geqslant 2$, $\sum_{\gamma\in\Gamma} h_k(\gamma, z)$ converges absolutely and uniformly on any compact subset of $\mathbb{H}^n$.*

Proof. Note that

$$\sum_{\gamma\in\Gamma} h_k(\gamma, z) = N(z-\bar{z})^{2k} B_k(z, z) \tag{2.2}$$

and then the result follows from Proposition 3. □

Lemma 4. *For any $M \in \Gamma$, we have*

$$h_k\big(M^{-1}\gamma M, z\big) = h_k(\gamma, Mz).$$

Proof. By a simple computation or see [Fr]. □

3. Proof of Theorem 1

Before we prove the theorem, we make the following observation.

Since $B_k(z, w)$ is a cusp form in z (by Proposition 3), we have

$$\begin{aligned} B_k(z, w) &= \sum_{i=1}^{J_k} \big\langle B_k(\cdot, w), f_{i,k}\big\rangle f_{i,k}(z) \\ &= C_k \sum_{i=1}^{J_k} \overline{f_{i,k}(w)} f_{i,k}(z) \quad \text{(by Proposition 4)}. \end{aligned}$$

Let $w = z$, then we obtain the identity

$$B_k(z, z) = C_k \sum_{i=1}^{J_k} \big|f_{i,k}(z)\big|^2, \tag{3.1}$$

where C_k is defined in (2.1).

Proof of Theorem 1. Let $\chi_A(z)$ denote the characteristic function of A on $\Gamma\backslash\mathbb{H}^n$. One can extend it (with the same notation) to $\mathbb{H}^n$ as a Γ-invariant function.

By (3.1) and (2.2),

$$\int_A d\mu_k = \frac{1}{J_k C_k} \int_A B_k(z,z)(Ny)^{2k}\, d\mu$$
$$= \frac{1}{(2i)^{2kn} J_k C_k} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) \sum_{\gamma\in\Gamma} h_k(\gamma,z)\, d\mu$$
$$= \frac{1}{(2i)^{2kn} J_k C_k} \Bigg[\sum_{\gamma=\pm 1} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma,z)\, d\mu$$
$$+ \sum_{\gamma\in\Gamma,\ \gamma \text{ is elliptic}} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma,z)\, d\mu$$
$$+ \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) \Bigg(\sum_{\gamma\in\Gamma,\ \gamma\neq\pm 1,\ \gamma \text{ is not elliptic}} h_k(\gamma,z) \Bigg) d\mu \Bigg].$$

We estimate the above three summation of integrals in the following cases.

Case 1. $\gamma = \pm 1$.

$$\int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma,z)\, d\mu = \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z)\, d\mu = \mu(A).$$

Case 2. For $\gamma \in \Gamma$ elliptic, let

$$\Gamma_\gamma = \{M \in \Gamma\colon\ M\gamma = \gamma M\} \quad (\text{the centralizer of } \gamma \text{ in } \Gamma)$$

and

$$[\gamma] = \{M^{-1}\gamma M\colon\ M \in \Gamma\}.$$

Also let Λ be a set of complete representatives of elliptic conjugate classes in Γ.

Remark. $|\Lambda| < \infty$ by Lemma 1.

$$\sum_{\gamma\in\Gamma,\ \gamma \text{ is elliptic}} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma,z)\, d\mu = \sum_{\gamma\in\Lambda} \sum_{\gamma'\in[\gamma]} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma',z)\, d\mu$$
$$= \sum_{\gamma\in\Lambda} \sum_{M\in\Gamma_\gamma\backslash\Gamma} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(M^{-1}\gamma M, z)\, d\mu.$$

Using Lemma 4 and unfolding, we have

$$\sum_{M\in\Gamma_\gamma\backslash\Gamma} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(M^{-1}\gamma M, z)\, d\mu = \int_{\Gamma_\gamma\backslash\mathbb{H}^n} \chi_A(z) h_k(\gamma,z)\, d\mu$$

$$= \frac{1}{|\Gamma_\gamma|} \int\limits_{\mathbb{H}^n} \chi_A(z) h_k(\gamma, z)\, d\mu$$

$$= \frac{1}{|\Gamma_\gamma|} \int\limits_{\mathbb{H}^n} \chi_A(z) \prod_{i=1}^{n} h_{k,i}(\gamma_i, z_i)\, d\mu$$

$$\leqslant \frac{1}{|\Gamma_\gamma|} \frac{1}{\mathrm{vol}(\Gamma\backslash\mathbb{H}^n)} \prod_{i=1}^{n} \int\limits_{\mathbb{H}} h_{k,i}(\gamma_i, z_i) \frac{dx_i\, dy_i}{y_i^2}$$

where

$$h_{k,i}(\gamma_i, z_i) = (z_i - \overline{z_i})^{2k} (\gamma_i z_i - \overline{z_i})^{-2k} j(\gamma_i, z_i)^{-2k}.$$

Remark. $h_{k,i}(M^{-1}\gamma_i M, z_i) = h_{k,i}(\gamma_i, Mz_i)$ for any $M \in SL(2,\mathbb{R})$.

Hence we may assume that each γ_i is of the form

$$\begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}, \quad \theta_i \neq 0, \pi.$$

For convenience, we drop the subscripts i in γ_i, z_i, θ_i, etc.

Now we make change of variables by using the *Cayley transform*

$$\mathbb{H} \to D \text{ (unit disc)}$$

$$z \mapsto w = \frac{z-i}{z+i}$$

and then use the polar coordinates $w = \rho e^{i\varphi}$ of the unit disc. It yields

$$\int\limits_{\mathbb{H}} \left| h_{k,i}(\gamma, z) \right| \frac{dx\, dy}{y^2} = 4 \int\limits_0^{2\pi} \int\limits_0^1 \frac{(1-\rho^2)^{2k-2}}{|1 - e^{i\beta}\rho^2|^{2k}} \rho\, d\rho\, d\varphi$$

$$= 4\pi \int\limits_0^1 \frac{(1-t)^{2k-2}}{|1-e^{i\beta}t|^{2k}}\, dt \quad (\text{where } \beta = 2\theta \neq 0, 2\pi).$$

- When $0 \leqslant t \leqslant k^{-1+\epsilon}$ $(0 < \epsilon < 1)$, it is easy to see that $\frac{1-t}{|1-e^{i\beta}t|} \leqslant 1$. Hence

$$\int\limits_0^{k^{-1+\epsilon}} \frac{(1-t)^{2k-2}}{|1-e^{i\beta}t|^{2k}}\, dt = \int\limits_0^{k^{-1+\epsilon}} \left(\frac{1-t}{|1-e^{i\beta}t|} \right)^{2k-1} \frac{1}{|1-e^{i\beta}t|^2}\, dt$$

$$\leqslant \int\limits_0^{k^{-1+\epsilon}} \frac{1}{|1-e^{i\beta}t|^2}\, dt \ll k^{-1+\epsilon}.$$

• When $k^{-1+\epsilon} \leqslant t \leqslant 1$, we have $(1-t)^2 < \frac{1}{4}$ for k sufficiently large and then $\frac{2t}{(1-t)^2} \geqslant \frac{2k^{-1+\epsilon}}{4} = \frac{1}{2}k^{-1+\epsilon}$. So

$$\frac{1-t}{|1-e^{i\beta}t|} = \frac{1}{|1+\frac{2t}{(1-t)^2}(1-\cos\beta)|^{1/2}} \ll \left(1+k^{-1+\epsilon}\right)^{-1/2}.$$

Hence

$$\int_{k^{-1+\epsilon}}^{1} \frac{(1-t)^{2k-2}}{|1-e^{i\beta}t|^{2k}}\,dt \ll \left[\left(1+k^{-1+\epsilon}\right)^{-1/2}\right]^{2k-2} = \left(1+k^{-1+\epsilon}\right)^{-k+1}.$$

Combining these estimates, we get

$$\sum_{\gamma\in\Gamma,\,\gamma\text{ is elliptic}} \int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z)h_k(\gamma,z)\,d\mu \ll \left(k^{-1+\epsilon}\right)^n.$$

Note that here the implicit constant only depends on ϵ.

Case 3. Let $\Gamma' = \Gamma \setminus (\{\pm 1\} \cup \{\gamma \in \Gamma\colon\ \gamma \text{ is elliptic}\})$.

Since $\sum_{\gamma\in\Gamma'} |h_3(\gamma,z)|$ converges uniformly on A (by Lemma 3) and $|h_3(\gamma,z)| < 1$ for all $z \in A$, $\gamma \in \Gamma'$ (by Lemma 2), there exists a constant $0 < \lambda < 1$ (dependent on A) such that $|h_3(\gamma,z)| < \lambda$ for all $z \in A$, $\gamma \in \Gamma'$. Hence

$$\begin{aligned}\int_{\Gamma\backslash\mathbb{H}^n} \chi_A(z)\left(\sum_{\gamma\in\Gamma'} h_k(\gamma,z)\right)d\mu &\leqslant \int_A \sum_{\gamma\in\Gamma'} \left|h_3(\gamma,z)\right|\left|h_3(\gamma,z)\right|^{\frac{k-3}{3}}\,d\mu \\ &\leqslant \int_A \sum_{\gamma\in\Gamma'} \left|h_3(\gamma,z)\right|\lambda^{\frac{k-3}{3}}\,d\mu \ll (\lambda_1)^k\end{aligned}$$

where $\lambda_1 = (\lambda)^3 < 1$.

From Cases 1–3 and using Shimizu's asymptotic formula (1.1) for J_k, Theorem 1 follows directly. □

4. Some remarks

Let Γ be a discrete subgroup of $SL(2,\mathbb{R})^n$ with finite co-volume which satisfies the irreducibility condition below and Assumption (F) on its fundamental domain.

Irreducibility condition: The restriction of each of the n projections

$$p_j : SL(2,\mathbb{R})^n \to SL(2,\mathbb{R}) \quad (1 \leqslant j \leqslant n)$$

to Γ is injective.

Assumption (F): Let κ_v $(1 \leqslant v \leqslant t)$ be a set of complete representatives of Γ-inequivalent cusp of Γ. For each v, take a $g_v \in SL(2, \mathbb{R})^n$ such that $g_v \kappa_v = \infty$ and put

$$U_v = \left\{ g_v^{-1} z \colon \prod_{i=1}^{n} \operatorname{Im}(z_i) > d_v,\ z = (z_1, \ldots, z_n) \right\}$$

where d_v is a suitably chosen positive number. Let $\Gamma_{\kappa_v} = \{\gamma \in \Gamma \colon\ \gamma\kappa_v = \kappa_v\}$ and let V_v be a fundamental domain of Γ_{κ_v} in U_v. Then Γ has a fundamental domain F of the form

$$F = F_0 \cup V_1 \cup \cdots \cup V_t$$

where F_0 is relatively compact in $\mathbb{H}^n$.

In this case, Shimizu's dimension formula (1.1) also holds for Γ [Sh]. Moreover, our propositions, lemmas and theorem in previous sections all remain true for Γ. In particular, for a non-zero ideal $\mathfrak{n}$ of $\mathcal{O}$, let

$$\Gamma_0(\mathfrak{n}) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathcal{O}) \colon\ c \equiv 0 \text{ modulo } \mathfrak{n} \right\}.$$

Then $\Gamma = \Gamma_0(\mathfrak{n})$ satisfies the irreducible condition and Assumption (F). Hence we have the following corollary:

Corollary 2. *For any compact subset $A \subset \Gamma_0(\mathfrak{n}) \backslash \mathbb{H}^n$ and any $0 < \epsilon < 1$, we have*

$$\int_A d\mu_k = \int_A d\mu + O_{\epsilon, A}\big(\big(k^{-1+\epsilon}\big)^n\big)$$

as $k \to \infty$.

Remark. Again the decay rate here is sharper than in [La], but the implicit constant depends on the compact subset A. In [La], the result is uniform.

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